# Final exam for Kwantumfysica 1 - 2004-2005

**Thursday 23 June 2005 14:00 - 17:00**

**<u>READ THIS FIRST</u>:**

- Clearly write your name and study number on each answer sheet that you use.
- On the first answer sheet, write clearly the total number of answer sheets that you turn in.
- Note that this exam has 4 questions, it continues on the backside of the papers!
- Start each question (number 1, 2, 3, 4) on a new answer sheet.
- The exam is open book. You are also allowed to use formula sheets etc.
- If it says "make a rough estimate", there is no need to make a detailed calculation, and making a simple estimate is good enough. If it says "calculate" or derive, you are supposed to present a full analytical calculation.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first.

**Useful formulas and constants:**

| | | |
|---|---|---|
| Electron mass | $m_e$ | $= 9.1 \cdot 10^{-31}$ kg |
| Electron charge | $-e$ | $= -1.6 \cdot 10^{-19}$ C |
| Planck's constant | $h$ | $= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs |
| Planck's reduced constant | $\hbar$ | $= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs |

Fourier relation between *x*-representation and *k*-representation of a state

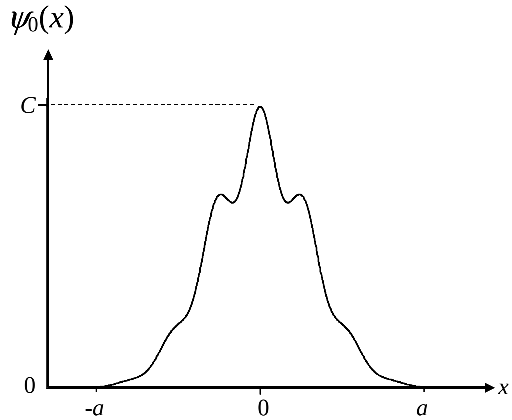
$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overline{\Psi}(k) e^{ikx}\, dk$$

$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx}\, dx$$

# Z.O.Z.

**Problem 1**

Consider a one-dimensional potential well with a flat bottom at $V(x) = 0$ for $x$ around $x = 0$, and $V(x) = \infty$ for $x > a$ and $x < -a$. For this system $a = 1$ nm. The well contains a single point particle with mass $m$. This system is prepared in the state $\psi_0(x)$, where its normalized wavefunction is as in the following sketch (note that this wavefunction can be represented by real values).




**a)** Make a *rough estimate* for the value of the constant $C$ in this plot, which sets the scale for the amplitude of the wavefunction (and make sure you give it the right unit).

**b)** Sketch the energy eigenfunctions $\varphi_1(x)$, $\varphi_2(x)$ and $\varphi_3(x)$ of this system that are associated with the three lowest energy eigenvalues $E_1$, $E_2$ and $E_3$ (also here, choose any arbitrary phase factors such that the eigenfunctions are represented by real values).

**c)** What is the parity of the state $\psi_0(x)$? Explain your answer.

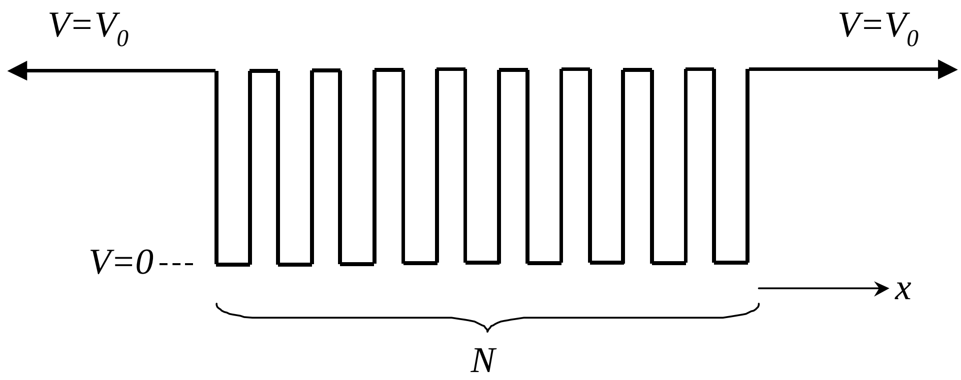
**d)** One has a technique to determine experimentally in which energy eigenstate the system is. With the system prepared in the state as in the sketch, one performs this measurement. Write down analytical expressions in the $x$-representation for the probabilities $W(E_1)$, $W(E_2)$ and $W(E_3)$ for finding in this experiment $E_1$, $E_2$ and $E_3$ (no need to work them out here)

**e)** As question d), but now explain which of the three probabilities $W(E_1)$, $W(E_2)$ and $W(E_3)$ is highest, and which is lowest. Explain how you get the answers.

**f)** As question c), but now make *rough estimates* for the values of $W(E_1)$, $W(E_2)$ and $W(E_3)$. Explain how you get the answers.

**Problem 2**

Consider a one-dimensional array of $N$ potential wells, formed by the potential $V(x)$ as in the following figure. The width of the wells is $a$, the width of the barriers between the wells is $b$.




**a)** Consider the case $N = 1$, with the width of that well $a = 0.1$ nm, and $V_0 = 3$ eV. This well contains a single particle with mass $m$. Calculate for which values of $m$ this system has 3 bound energy eigenstates.

**b)** **<u>NOW READ QUESTION c) FIRST</u>**. Consider the case $N = 2$. Assume that the system contains a particle with the same mass as considered for question a). For $a = 0.1$ nm, $b = 0.1$ nm and $V_0 = 3$ eV, the tunnel coupling $T_0$ between the ground states of the left and the right well is $T_0 = 0.1$ eV. (Note that we mean here the ground states for the case that each well is not yet coupled to another well.) Make a rough sketch of the spectrum (as a function of $b$) of the bound energy eigenstates for a particle in this double well system, for the range $b = 0$ nm to $b = 10$ nm.

**c)** Add to the sketch of the spectrum of question b), several of the unbound energy eigenstates. Sketch only the ones with the lowest energy eigenvalues. Put labels in the sketch to point out which energy eigenvalues are bound states, and which energy eigenvalues are unbound states.

**d)** What is the level spacing between the unbound energy eigenvalues of question c)?

**e)** Now consider the case where $N$ is a very large number. Repeat question b) for this case.

**f)** Give an example of a real physical situation where the model system considered in e) is relevant. Explain your answer.

**Z.O.Z.**

**Problem 3**

**Note: where possible, use Dirac notation for solving this problem.** Consider a one-dimensional, mechanical harmonic oscillator system, for a particle with mass $m$. The Hamiltonian of this system is

$$\hat{H} = \hbar\omega_0\left(\hat{N} + \tfrac{1}{2}\right).$$

Here $\omega_0$ is the natural frequency of the system, and $\hat{N}$ the number operator (with eigenvalues $n$ and eigenstates $|\varphi_n\rangle$ ) for this system.

**a)** Write the Hamiltonian in terms of annihilation and creation operators.

**b)** Write the Hamiltonian in terms of position and momentum operators.

**c)** The system can be prepared in one of the two following states:

$$|\Psi_1\rangle = i\sqrt{\tfrac{1}{2}}|\varphi_0\rangle + \sqrt{\tfrac{3}{8}}|\varphi_1\rangle - i\sqrt{\tfrac{1}{8}}|\varphi_3\rangle$$
$$|\Psi_2\rangle = \sqrt{\tfrac{1}{3}}|\varphi_0\rangle + i\sqrt{\tfrac{1}{3}}|\varphi_2\rangle - i\sqrt{\tfrac{1}{3}}|\varphi_4\rangle$$

Show that these two states are normalized.

**d)** Calculate the inner product between $|\Psi_1\rangle$ and $|\Psi_2\rangle$.

**e)** The particle in this system has a charge $q$, and feels the harmonic oscillator potential because it is attracted to the lowest point in the potential by an electrical force. If this charge moves, the system will emit electromagnetic radiation. Assume that this position depended electric field that causes the radiation can be describe by an operator $\hat{D}$, and that the system at time $t = 0$ is prepared in state the $|\Psi_2\rangle$. Show with a calculation whether, and if so at what frequencies, this system will in this case emit electromagnetic radiation at times right after $t = 0$.

**Problem 4**

A free particle is at some moment in time in a state for which the wavefunction can be described as

$$\Psi(x) = \begin{cases} C \quad , & |x| \le a. \\ 0 \ , & |x| > a. \end{cases}$$

**a)** Derive the wavefunction of this particle in $k$-representation for this moment in time.

**b)** Describe in words (if you like also use formulas) how the wavefunction in $k$-representation will develop in time, for times after this moment.